Fermion zero mode and superfluid weight

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We propose one possible mechanism for deconfinement based on an SU(2) slave-boson theory. Resorting to an effective field theory approach, we show that introduction of an isospin interaction potential gives rise to a fermion zero mode in an instanton-hedgehog configuration. As a result, meron-type vortices are allowed. We demonstrate how emergence of such vortices results in the doping-independent decreasing ratio of superfluid weight.

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Study on strongly correlated electrons opened a new window so called gauge theory in modern condensed matter physics beyond the Fermi liquid theory and Landau-Ginzburg-Wilson framework of "classical" condensed matter physics. When interactions are strong enough compared with kinetic energy, an infinite interaction limit can be a good starting point. The presence of such a large energy scale gives rise to a constraint in dynamics of electrons. In addition, interesting physics now arises from the kinetic-energy contribution in the restricted Hilbert space, and "non-local" order parameters or more carefully, link variables instead of on-site ones in lattice models appear as important low energy collective degrees of freedom. These link variables can be formulated as gauge fields, and gauge theory arises naturally for dynamics of strongly correlated electrons.

Slave-boson approach has been one of the canonical frameworks for study of strongly correlated electrons. In particular, a doped Mott insulator problem was formulated in the slave-boson context,[1] where strong repulsive interactions cause so called the single occupancy constraint naturally imposed in the slave-boson representation, and link variables arise as collective "order parameter" excitations formulated as gauge fields. U(1) slave-boson gauge theory has been enjoyed both intensively and extensively for the doped Mott insulator problem.

It will be not only fair but also true to say that this gauge theoretical framework has explained many kinds of aspects associated with high T_c cuprates such as phase diagram, thermodynamics, transport, spin dynamics, and etc.[1] However, such a theoretical structure seems to have fundamental difficulty for physics of high T_c cuprates. One is about the presence of coherent electron-like excitations near nodal directions in the normal state of high T_c cuprates.[2] Although the slave-boson framework can recover Fermi liquid via condensation of a bosonic charge degree of freedom (holon), such coherent electron excitations disappear in an uncondensed phase as the above. This problem was argued to be related with so called confinement in gauge theory,[3] one of the notorious problems in theoretical physics, and it requires deeper understanding of instanton physics in gauge fluctuations.

The other is associated with the doping-independent decreasing ratio of superfluid weight.[4] Although this problem seems to be simple compared with the first one, information in superfluid weight has important physical implication since it reflects d-wave superconductivity emerging from a doped Mott insulator. Unfortunately, the U(1) slave-boson framework cannot explain this doping independence owing to charge renormalization of nodal quasiparticles although it gives the correct zero temperature superfluid density proportional to hole concentration.[5, 6]

The second problem has motivated Wen and Lee to propose an SU(2) slave-boson formulation, which extends the U(1) slave-boson theory to include fluctuations between nearly degenerate U(1) mean-field states, well applicable in underdoped regions.[1] In the SU(2) slave-boson representation an additional holon excitation called b_2 boson appears to take such fluctuations into account. Based on this formulation, they demonstrated how one can find the doping independent decreasing ratio of superfluid weight. Assuming confinement of spinons and holons into electrons, they could show that such charge renormalization does not occur in the SU(2) slave-boson formulation.[6, 7]

In this paper we revisit this issue based on one possible deconfinement scenario. Our main observation is that the SU(2) framework can give rise to an isospin interaction potential via gauge fluctuations. Actually, such an isospin interaction was introduced by Wen and Lee.[8] They studied its perturbation effect, and found that Fermi segments of electron excitations away from half filling can appear owing to the presence of such a term although spinons are still at half filling in the SU(2) slave-boson formulation.

In this paper we consider its non-perturbation effect. It is important to notice that the SU(2) slave-boson gauge theory allows meron-type vortices, and their tunnelling events from up meron vortices to down ones are associated with instanton excitations of U(1) gauge fields. In other words, a monopole configuration of U(1) gauge fields induces a hedgehog pattern of holon isospins. As a result, the effective field theory with an isospin interaction term has a similar form with the SU(2) gauge theory well studied by Jackiw and Rebbi, where 't Hooft monopole excitations are allowed and a fermion zero mode exists due to the presence of an isospin interaction.[9] Actually, we find a fermion zero

mode from an explicit calculation in two space and one time dimensions $[(2+1)D]$. Thus, instanton excitations are suppressed and deconfinement of spinons and holons is realized. This scenario was pursued in compact QED_3 without an isospin interaction by Marston, but the zero mode was proven not to exist in such an effective field theory.[10]

Although our zero mode scenario is appealing, we should confess that the fermion zero mode can become unstable because there is no gap to protect the mode. We will discuss this possibility in more detail.

Based on this zero-mode scenario to allow meron vortices, we discuss superfluid weight. Resorting to a dual Lagrangian for meron vortices, we find that charge renormalization does not occur in a certain limit, thus resulting in the doping independent decreasing ratio of superfluid weight.

Dynamics of doped holes in the antiferromagnetically correlated spin background is described by the t-J Hamiltonian

$$H = -t\sum_{\langle ij\rangle}(c_{i\sigma}^\dagger c_{j\sigma} + H.c.) + J\sum_{\langle ij\rangle}(\vec{S}_i\cdot\vec{S}_j - \frac{1}{4}n_i n_j). \tag{1}$$

Introducing an SU(2) slave-boson representation for an electron field

$$\begin{aligned} c_{i\uparrow} &= \frac{1}{\sqrt{2}}h_i^\dagger\psi_{i+} = \frac{1}{\sqrt{2}}(b_{i1}^\dagger f_{i1} + b_{i2}^\dagger f_{i2}^\dagger), \\ c_{i\downarrow} &= \frac{1}{\sqrt{2}}h_i^\dagger\psi_{i-} = \frac{1}{\sqrt{2}}(b_{i1}^\dagger f_{i2} - b_{i2}^\dagger f_{i1}^\dagger), \end{aligned} \tag{2}$$

where $\psi_{i+} = \begin{pmatrix} f_{i1} \\ f_{i2}^\dagger \end{pmatrix}$ and $\psi_{i-} = \begin{pmatrix} f_{i2} \\ -f_{i1}^\dagger \end{pmatrix}$ are SU(2) spinon-spinors and $h_i = \begin{pmatrix} b_{i1} \\ b_{i2} \end{pmatrix}$ is holon-spinor, one can rewrite the t-J model in terms of these fractionalized excitations with hopping and pairing fluctuations

$$\begin{aligned} & L = L_0 + L_s + L_h, \qquad L_0 = J_r\sum_{\langle ij\rangle}\mathbf{tr}[U_{ij}^\dagger U_{ij}], \\ & L_s = \frac{1}{2}\sum_i \psi_{i\alpha}^\dagger(\partial_\tau - ia_{i0}^k\tau_k)\psi_{i\alpha} \\ & +J_r\sum_{\langle ij\rangle}(\psi_{i\alpha}^\dagger U_{ij}\psi_{j\alpha} + H.c.), \\ & L_h = \sum_i h_i^\dagger(\partial_\tau - \mu - ia_{i0}^k\tau_k)h_i \\ & +t_r\sum_{\langle ij\rangle}(h_i^\dagger U_{ij}h_j + H.c.), \end{aligned} \tag{3}$$

where the SU(2) matrix field is $U_{ij} = \begin{pmatrix} -\chi_{ij}^\dagger & \eta_{ij} \\ \eta_{ij}^\dagger & \chi_{ij} \end{pmatrix}$, and $J_r = \frac{3J}{16}$ and $t_r = \frac{t}{2}$ are redefined couplings.[1] Since this decomposition representation enlarges the original electron Hilbert space, constraints are introduced via Lagrange multiplier fields a_{i0}^k with $k = 1, 2, 3$.

In the SU(2) formulation Wen and Lee choose the staggered flux gauge[1]

$$U_{ij}^{SF} = -\sqrt{\chi^2+\eta^2}\tau_3\exp[i(-1)^{i_x+i_y}\Phi\tau_3] \tag{4}$$

with a phase $\Phi = \tan^{-1}\Big(\frac{\eta}{\chi}\Big)$. Although the staggered flux ansatz breaks translational invariance, this formal symmetry breaking is restored via SU(2) fluctuations between nearly degenerate U(1) mean-field states. For example, one possible U(1) ground state, the d-wave pairing one $U_{ij}^{dSC} = -\chi\tau_3 + (-1)^{i_y+j_y}\eta\tau_1$ can result from the SU(2) rotation $U_{ij}^{dSC} = W_i U_{ij}^{SF} W_j^\dagger$ with an SU(2) matrix $W_i = \exp\Big\{i(-1)^{i_x+i_y}\frac{\pi}{4}\tau_1\Big\}$. Then, our starting point becomes the following effective Lagrangian

$$\begin{aligned} L_{SF} &= \frac{1}{2}\sum_i \psi_{i\alpha}^\dagger(\partial_\tau - ia_{i0}^3\tau_3)\psi_{i\alpha} \\ &+J_r\sum_{\langle ij\rangle}(\psi_{i\alpha}^\dagger U_{ij}^{SF}e^{ia_{ij}^3\tau_3}\psi_{j\alpha} + H.c.) \\ &+\sum_i h_i^\dagger(\partial_\tau - \mu - ia_{i0}^3\tau_3)h_i \\ &+t_r\sum_{\langle ij\rangle}(h_i^\dagger U_{ij}^{SF}e^{ia_{ij}^3\tau_3}h_j + H.c.) \\ &+J_r\sum_{\langle ij\rangle}\mathbf{tr}[U_{ij}^{SF\dagger}U_{ij}^{SF}], \end{aligned} \tag{5}$$

where we have introduced only one kind of gauge field a_μ^3 as important low energy fluctuations since other two ones, a_μ^1 and a_μ^2 are gapped due to Anderson-Higgs mechanism in the staggered flux phase.

One can derive the following effective field theory from the staggered flux ansatz of the SU(2) slave-boson theory Eq. (5),

$$\begin{aligned} \mathcal{L}_{SF} &= \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu^3\tau_3)\psi + \frac{1}{2g^2}(\epsilon_{\mu\nu\gamma}\partial_\nu a_\gamma^3)^2 \\ &+x\mathbf{z}^\dagger(\partial_\tau - ia_0^3\tau_3 - iA_0)\mathbf{z} + \frac{1}{u_h}|\mathbf{z}^\dagger(\partial_\tau - ia_0^3\tau_3 - iA_0)\mathbf{z}|^2 \\ &+\frac{x}{2m_b}|(\partial_i - ia_i^3\tau_3 - iA_i)\mathbf{z}|^2 \\ &+x^2J\Big[\frac{4}{c_1}|z_1|^2|z_2|^2 + \frac{1}{c_2}(|z_1|^2-|z_2|^2)^2\Big]. \end{aligned} \tag{6}$$

Dirac structure for spinon dynamics[11] results from the staggered flux gauge in the SU(2) slave-boson theory, where ψ is an 8 component spinor and Dirac gamma matrices are $\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$, $\gamma_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}$, and $\gamma_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}$. a_μ^3 is the remaining massless U(1) gauge field as an important low energy degree of freedom in the staggered flux phase. We have introduced a finite bare gauge charge g in the long wave-length limit. It is

important to understand that spinons are still at half filling even away from half filling in the SU(2) formulation. The single-occupancy constraint in the SU(2) representation is given by $f_{i1}^\dagger f_{i1} + f_{i2}^\dagger f_{i2} + b_{i1}^\dagger b_{i1} - b_{i2}^\dagger b_{i2} = 1$. Thus, if the condition of $\langle b_{i1}^\dagger b_{i1}\rangle = \langle b_{i2}^\dagger b_{i2}\rangle = \frac{x}{2}$ with hole concentration x is satisfied, we see $\langle f_{i1}^\dagger f_{i1} + f_{i2}^\dagger f_{i2}\rangle = 1$, i.e., spinons are at half filling. As a result, a chemical potential term does not arise in the spinon sector. Actually, this was demonstrated for the staggered flux phase in the mean-field analysis of the SU(2) slave-boson theory.[8]

Holon dynamics at low energies is described by CP^1 gauge theory of the O(3) nonlinear σ model[12] with a Berry phase contribution (the first term in the holon sector) arising from finite density of holons $\mathbf{z} = \begin{pmatrix} z_+ \\ z_- \end{pmatrix}$ while z_+ gauge charge is opposite to z_- one. u_h is associated with compressibility for holons, and $m_b \sim 1/t$ is bare band mass. The last two terms in the holon sector represent anisotropy contributions of holon isospins, given by $\vec{I}_{hi} = z_{i\alpha}^\dagger \vec{\tau}_{\alpha\beta} z_{i\beta} = (\sin\theta_i\cos\phi_i, \sin\theta_i\sin\phi_i, \cos\theta_i)$, where c_1 and c_2 are positive numerical constants, possibly arising from short-distance fermion fluctuations. In the SU(2) slave-boson theory each phase can be identified with this isospin configuration, where the staggered flux phase is characterized by $\langle \vec{I}_h\rangle = 0$ while the d-wave pairing state is expressed as $\langle I_h^z\rangle = 0$ and $\langle I_h^{x(y)}\rangle \neq 0$. The above effective field theory describes fluctuations between nearly degenerate U(1) mean-field states via isospin fluctuations. The last two terms favor an easy plane when $c_1 > c_2$ expected away from half filling.

In the easy plain limit one can represent the holon spinor as $z_\sigma = e^{i\phi_\sigma}$. Then, the above effective field theory becomes

$$\begin{aligned}\mathcal{L} &= \bar{\psi}_\sigma \gamma_\mu (\partial_\mu - i\sigma a_\mu)\psi_\sigma \\ &+ \frac{1}{u_h}\Big(\partial_\tau \phi_\sigma - \sigma a_0 - A_0 + i\frac{u_h}{2}x\Big)^2 \\ &+ \frac{x}{2m_b}(\partial_i\phi_\sigma - \sigma a_i - A_i)^2 + \frac{1}{2g^2}(\epsilon_{\mu\nu\gamma}\partial_\nu a_\gamma)^2, \quad (7)\end{aligned}$$

where $\pm$ are associated with the SU(2) space and the superscript 3 in the gauge field is omitted.

We first consider an infinite bare gauge coupling $g \to \infty$, where U(1) gauge fields can be integrated out exactly resulting in confinement.[5, 13] Shifting gauge fields as $a_0 \to a_0 - \partial_\tau\phi_- + A_0 - i\frac{u_h}{2}x$ and $a_i \to a_i - \partial_i\phi_- + A_i$, and introducing field variables of $c_\sigma = e^{i\sigma\phi_-}\psi_\sigma$ and $\phi_c = \phi_+ + \phi_-$, one can perform integration of U(1) gauge fields and find an effective field theory for d-wave superconductivity emerging from a doped Mott insulator in the confinement limit

$$\begin{aligned}\mathcal{L} &= \bar{c}_\sigma\gamma_\mu\Big(\partial_\mu - \frac{i}{2}\sigma\partial_\mu\phi_c\Big)c_\sigma \\ &+ \frac{u_h}{8}(\sigma\bar{c}_\sigma\gamma_0 c_\sigma)^2 + \frac{m_b}{4x}(\sigma\bar{c}_\sigma\gamma_i c_\sigma)^2 \\ &+ \frac{1}{2u_h}\Big(\partial_\tau\phi_c - 2A_0 + iu_h x\Big)^2 + \frac{x}{4m_b}\Big(\partial_i\phi_c - 2A_i\Big)^2 (8)\end{aligned}$$

It should be noted that all dynamic variables are gauge singlets, thus gauge invariance associated with a_μ is satisfied automatically. As shown in this field theory, charge renormalization does not occur although the phase stiffness is proportional to hole concentration implying that this superconductivity results from a doped Mott insulator. This is in contrast with the U(1) formulation, where electric charge of quasiparticles is proportional to hole concentration.[5, 6] As a result, we obtain the following expression for superfluid weight $\rho_s(x,T) = \rho_s(x) - cT$, where the decreasing ratio $\frac{d\rho_s(x,T)}{dT} = -c$ is doping independent. Actually, this confinement scenario was investigated previously, but in a different version.[6]

If the bare gauge charge is finite, one cannot perform integration of gauge fluctuations exactly. Considering that spinon contributions give rise to the following renormalized gauge dynamics $\frac{N}{16}(\partial\times a)\frac{1}{\sqrt{-\partial^2}}(\partial\times a)$ with the flavor number N, and performing duality transformation for holon vortices and instantons, we find an effective Lagrangian

$$\begin{aligned}\mathcal{L}_{dual} &= |(\partial_\mu - ic_\mu^\sigma)\Phi_\sigma|^2 + m_v^2|\Phi_\sigma|^2 + V_{eff}(|\Phi_\sigma|) \\ &+ \frac{u_h}{4}[(\partial\times c_\sigma)_\tau - x]^2 + \frac{m_b}{2x}(\partial\times c_\sigma)_i^2 \\ &+ \frac{4}{N}(\partial_\mu\varphi - c_+ + c_-)\sqrt{-\partial^2}(\partial_\mu\varphi - c_+ + c_-) \\ &- y_m(e^{i\varphi}\Phi_+^\dagger\Phi_- + H.c.). \quad (9)\end{aligned}$$

$\Phi_\pm$ represent holon vortices, and $c_\mu^\pm$ are vortex gauge fields associated with their long-range interactions. φ is a magnetic potential field associated with instanton excitations of U(1) gauge fields a_μ, and y_m is an instanton fugacity. In particular, the last term describes tunnelling events between + and − meron vortices, where instanton excitations are summed in the dilute approximation.

Eq. (9) has been studied intensively in the context of quantum antiferromagnetism,[14, 15] where fermion excitations are gapped thus ignored in the low energy limit, allowing a simpler expression $\mathcal{L}_{dual} = |(\partial_\mu - ic_\mu)\Phi_\sigma|^2 + m_v^2|\Phi_\sigma|^2 + V_{eff}(|\Phi_\sigma|) + \frac{u_h}{2}[(\partial\times c)_\tau - x]^2 + \frac{m_b}{x}(\partial\times c)_i^2 - y_m([\Phi_+^\dagger\Phi_-]^n + H.c.)$ with $x = 0$ and integer n. Here, the additional multiplication with n is argued to arise from a Berry phase contribution of gauge field. Although gapless fermion excitations renormalize gauge dynamics, the instanton-induced term will be relevant in both staggered flux and superconducting phases, i.e., away from quantum criticality. On the other hand, such instanton fluctuations are claimed to be irrelevant at the quantum critical point even in the $n = 1$ case, allowing meron-type vortices at the quantum critical point.[15]

We propose a deconfinement mechanism which has nothing to do with critical fluctuations[14, 15, 16]. Mentioned in the introduction, a fermion zero mode can emerge to suppress instanton effects if the effective field theory Eq. (6) is modified slightly. It has been argued that gauge fluctuations of time components can induce an isospin interaction potential.[8] Introducing an isospin interaction term $\mathcal{L}_I = U_I\bar{\psi}(\vec{I}_h\cdot\vec{\tau})\psi$ with its coupling

strength U_I, we obtain an equation of motion for Dirac fermions in $(2+1)D$

$$\gamma_\mu(\partial_\mu - i a^{cl}_{3\mu}\tau_3)\psi + U_I(\vec{I}^{cl}_h \cdot \vec{\tau})\psi = E\psi, \qquad (10)$$

where E is an eigen value.

A full procedure should be as follows. Starting from Eq. (6) with the isospin coupling term, we derive two equations of motion for holons and spinons, respectively. Considering an instanton configuration in the gauge potential, we solve the holon sector and find its corresponding holon configuration. Remember that the holon sector is exactly the same as the CP^1 gauge theory if the contribution from finite density of holons is neglected, i.e., the linear time-derivative term. The σ model study has shown that an instanton potential gives rise to a hedgehog configuration of spins in $(2+1)D$.[17] Actually, this contribution is reflected in the instanton-induced hedgehog term of the dual vortex Lagrangian, i.e., $-y_m(\Phi_+^\dagger\Phi_- + H.c.)$. In the present paper we do not solve the holon sector and assume the presence of such a solution. As far as hole concentration is not too large, we expect that our zero mode scenario may be applicable at least since the contribution from finite density of holons (the Berry phase term in the holon sector) would not spoil such a configuration.[18]

Based on this discussion, we solve the fermion part Eq. (10). The presence of an isospin interaction term reminds us of the SU(2) gauge theory in terms of massless Dirac fermions and adjoint Higgs fields interacting via SU(2) gauge fields, where topologically nontrivial stable excitations called 't Hooft-Polyakov monopoles are allowed. Jackiw and Rebbi have shown that the Dirac equation with isospin couplings has a fermion zero mode in a 't Hooft-Polyakov monopole potential.[9] Actually, we see that such a fermion zero mode exists indeed in our effective field theory. Existence of a fermion zero mode allows deconfinement of spinons and holons.

Considering an instanton configuration $a^{cl}_{3\mu} = a(s)\epsilon_{3\nu\mu}x_\nu$ with $a(s) \sim \frac{1}{s^2}$ for $s \to \infty$ where $s = \sqrt{\tau^2 + x^2 + y^2}$ and its corresponding isospin hedgehog configuration $I^{cl}_{h\mu} = \Phi(s)x_\mu$ with $\Phi(s) \sim \frac{1}{s}$ for $s \to \infty$, and inserting the 4 component spinor $\psi_n = \begin{pmatrix} \chi_n^+ \\ \chi_n^- \end{pmatrix}$ with an isospin index $n = 1, 2$ into Eq. (10), where $\chi_n^\pm$ represent 2 component spinors, we obtain

$$\begin{aligned}
&\sigma^3_{ij}\partial_\tau\chi^\pm_{jn} + \sigma^1_{ij}\partial_x\chi^\pm_{jn} + \sigma^2_{ij}\partial_y\chi^\pm_{jn} \\
&+ i a(s) y \sigma^1_{ij}\chi^\pm_{jm}\tau^3_{mn} - i a(s) x \sigma^2_{ij}\chi^\pm_{jm}\tau^3_{mn} \\
&\pm U_I\Phi(s)\chi^\pm_{im}(x_\mu\tau^T_\mu)_{mn} = 0. \qquad (11)
\end{aligned}$$

Here, the zero mode condition $E = 0$ is utilized. Observing the fact that discrimination between the isospin space and Dirac one disappears in the above expression, one can replace the $\vec{\tau}$ matrix with $\vec{\sigma}$.

Inserting the following expression $\chi^\pm_{jn} = \mathcal{M}^\pm_{jm}\sigma^3_{mn}$ with $\mathcal{M}^\pm_{jm} = g^\pm\delta_{jm} + g^\pm_\mu\sigma^\mu_{jm}$ into the above, where any 2×2 matrix $\mathcal{M}^\pm_{jm}$ can be represented with unit and Pauli matrices, we find

$$\begin{aligned}
&[\partial_\tau \pm U_I\Phi(s)\tau]g^\pm + i[\partial_x - a(s)x \pm U_I\Phi(s)x]g_2^\pm \\
&-i[\partial_y - a(s)y \mp U_I\Phi(s)y]g_1^\pm = 0, \\
&-[\partial_\tau \mp U_I\Phi(s)\tau]g_1^\pm + [\partial_x + a(s)x \pm U_I\Phi(s)x]g_3^\pm \\
&+i[\partial_y + a(s)y \pm U_I\Phi(s)y]g^\pm = 0, \\
&-[\partial_\tau \mp U_I\Phi(s)\tau]g_2^\pm - i[\partial_x + a(s)x \mp U_I\Phi(s)x]g^\pm \\
&+[\partial_y + a(s)y \mp U_I\Phi(s)y]g_3^\pm = 0, \\
&[\partial_\tau \pm U_I\Phi(s)\tau]g_3^\pm + [\partial_x - a(s)x \mp U_I\Phi(s)x]g_1^\pm \\
&+[\partial_y - a(s)y \pm U_I\Phi(s)y]g_2^\pm = 0. \qquad (12)
\end{aligned}$$

As a result, we find a zero mode equation

$$\begin{aligned}
&[\partial_\tau + U_I\Phi(s)\tau]g_1^- = 0, \\
&[\partial_x - a(s)x + U_I\Phi(s)x]g_1^- = 0, \\
&[\partial_y - a(s)y + U_I\Phi(s)y]g_1^- = 0, \qquad (13)
\end{aligned}$$

yielding one normalizable fermion zero mode $g_1^- \sim \exp\Big[-\int d\tau U_I\Phi(s)\tau\Big]\exp\Big[\int d\vec{r}\cdot\vec{r}\Big(a(s) - U_I\Phi(s)\Big)\Big]$. We note that this solution is basically the same as that of the SU(2) gauge theory by Jackiw and Rebbi.[9]

More fundamentally, the presence of such a fermion zero mode coincides with an index theorem,[3] stating that difference between the number of a zero mode with a left chirality and that with a right chirality is the same as a topological charge of the vacuum state. Since the topological vacuum charge is one, only one zero mode with a left chirality $-$ is found indeed.

Integrating out Dirac fermions, we see the partition function of instanton contributions as follows $Z_M = \exp\Big[N\mathbf{tr}\ln\Big\{\gamma_\mu(\partial_\mu - i a^{cl}_{3\mu}\tau_3) + U_I(\vec{I}^{cl}_h\cdot\vec{\tau})\Big\}\Big]$. Since the eigen value of the argument matrix is zero, the partition function vanishes, implying that single instanton excitations will be suppressed. This serves one possible deconfinement mechanism[19] that has nothing to do with critical fluctuations.

The final expression of our effective field theory becomes in the easy plane limit $c_1 > c_2$

$$\begin{aligned}
\mathcal{L}_{ZM} &= \bar{\psi}_\sigma\gamma_\mu(\partial_\mu - i\sigma a_\mu)\psi_\sigma + U_I\bar{\psi}(\vec{I}_h\cdot\vec{\tau})\psi \\
&+ \frac{1}{u_h}\Big(\partial_\tau\phi_\sigma - \sigma a_0 - A_0 + i\frac{u_h}{2}x\Big)^2 \\
&+ \frac{x}{2m_b}(\partial_i\phi_\sigma - \sigma a_i - A_i)^2 + \frac{1}{2g^2}(\epsilon_{\mu\nu\gamma}\partial_\nu a_\gamma)^2, \quad (14)
\end{aligned}$$

where U(1) gauge field is now noncompact. $\pm$ represent an isospin index. Since a non-perturbation effect of the isospin coupling term is already introduced in this expression, it will give a perturbation contribution such as a chemical potential term for nodal quasiparticles in the normal state.

Performing the duality transformation where perturbation effects of the isospin term are ignored, we find the

vortex Lagrangian

$$\mathcal{L}_{dual} = |(\partial_\mu - ic^\sigma_\mu)\Phi_\sigma|^2 + m_v^2|\Phi_\sigma|^2 + V_{eff}(|\Phi_\sigma|) + \frac{u_h}{4}[(\partial \times c_\sigma)_\tau - x]^2 + \frac{m_b}{2x}(\partial \times c_\sigma)_i^2 - i(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)(c^+_\mu - c^-_\mu) - i(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)(c^+_\mu + c^-_\mu) + \bar{\psi}_\sigma\gamma_\mu(\partial_\mu - i\sigma a_\mu)\psi_\sigma + \frac{1}{2g^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2. \quad (15)$$

Notice that the vortex-tunnelling term does not appear. As a result, meron vortices are allowed.

To discuss superfluid weight, we consider the superconducting phase characterized by $\langle \Phi_\sigma \rangle = 0$. Then, Eq. (15) reads

$$\mathcal{L}_{eff} = \frac{1}{2\rho_\sigma}(\epsilon_{\mu\nu\lambda}\partial_\nu c^\sigma_\lambda)^2 - i(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)(c^+_\mu - c^-_\mu) - i(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)(c^+_\mu + c^-_\mu) + \bar{\psi}_\sigma\gamma_\mu(\partial_\mu - i\sigma a_\mu)\psi_\sigma + \frac{1}{2g^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2, \quad (16)$$

where $\rho_\pm$ are stiffness parameters for $\pm$ holon fields, respectively.

Integrating out both vortex gauge fields $c^\pm_\mu$ and slave-boson U(1) gauge fields a_μ, we find an effective Lagrangian

$$\mathcal{L}_{eff} = \bar{\psi}_\sigma\gamma_\mu\Big\{\partial_\mu - i\Big(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\Big)\sigma A_\mu\Big\}\psi_\sigma + \frac{1}{2(\rho_+ + \rho_-)}(\sigma\bar{\psi}_\sigma\gamma_\mu\psi_\sigma)^2 + \frac{2\rho_+\rho_-}{\rho_+ + \rho_-}A_\mu^2. \quad (17)$$

This expression is quite interesting since charge renormalization of nodal quasiparticles does not occur if we assume $\rho_+ >> \rho_-$. Actually, this ansatz seems to be reasonable since b_+ holons exist in the U(1) formulation but b_- holons do not, reflecting reduction of the SU(2) symmetry down to U(1) away from half filling. In this limit we find the effective field theory $\mathcal{L}_{eff} = \bar{\psi}_\sigma\gamma_\mu(\partial_\mu - i\sigma A_\mu)\psi_\sigma + \frac{1}{2\rho_+}(\sigma\bar{\psi}_\sigma\gamma_\mu\psi_\sigma)^2 + 2\rho_- A_\mu^2$. An additional condition $\rho_- \propto x$ is necessary in order to obtain $T_c \propto x$ from the superfluid formulation. Despite this unsatisfactory point, we can see now why the superconducting transition temperature is not so high compared with the prediction of the U(1) slave-boson theory.[6] We note $T_c \propto \rho_+$ in the U(1) formulation, but $T_c \propto \rho_-$ in the SU(2) one. $\rho_+ >> \rho_-$, i.e., the contribution from SU(2) fluctuation explains this.

Several remarks are in order. First, one cautious theorist may suspect stability of the zero mode solution since there is expected to be no gap for spinons that remain at half-filling. In this respect the present situation for existence of a fermion zero mode is different from that discussed in Ref. [19] for antiferromagnetism, where the zero mode is a mid-gap state due to the presence of antiferromagnetism. At present, we cannot exclude that such a "gapless" zero mode may be unstable via quantum fluctuations. In this case the fermion-zero-mode mechanism for deconfinement is not applicable, and another mechanism associated with quantum criticality may be available.[14, 15, 16] Second, one may consider that the ad hoc limit $\rho_+ >> \rho_-$ introduced for explanation of the temperature dependence of the superfluid density is not consistent with the condition for existence of a fermion zero mode, that is, $\langle b^\dagger_{i1}b_{i1}\rangle = \langle b^\dagger_{i2}b_{i2}\rangle = \frac{x}{2}$ with hole concentration x, satisfied in the staggered flux phase. An important point is that ρ_+ and ρ_- are not the same as the density of each boson, respectively. As far as we know, this problem of interacting bosons with gauge fluctuations is not fully understood yet. Each holon density will be different from each superfluid density, thus the condition of $\rho_+ >> \rho_-$ should be regarded as another one, not inconsistent with the zero mode scenario. Third, one can propose a fermion zero mode localized in the meron vortex configuration instead of the hedgehog configuration. Actually, this is possible. Then, such meron vortices can acquire fermionic quantum numbers, for example spin.[9] Furthermore, their statistics may turn into fermions.[20] This rich possibility may open an important direction for study of high T_c cuprates.

One may ask whether the present zero mode scenario is consistent with the recent scanning tunnelling microscopy (STM) data[21] indicating columnar modulation in local density of states (LDOS). Unfortunately, we cannot say anything about the structure of vortices since vortices are taken into account as point particles in our effective field theory approach. What we can say is that tunnelling events between meron vortices can be suppressed via the fermion zero mode, implying that the SU(2) meron vortex will have the staggered flux core.[22] Actually, LDOS in the staggered-flux vortex core has been discussed in Ref. [23], but a direct comparison with the recent data is not clear. One possible scenario is as follows. In the staggered vortex core a chemical potential term arises for Dirac fermions since the density of b_1 bosons is different from that of b_2 ones. Then, hole pockets are allowed inside the vortex core. In this situation one may argue that quasiparticle scattering events near Dirac nodes can give rise to such density modulation, intensively discussed before.[24] This argument differs from the Cooper pair density wave as a possible Berry phase effect.[25]

In this paper we proposed one mechanism how meron-type vortices can appear beyond confinement based on a field theory approach, where such vortices are taken to be point particles although they have complex structures at short wave length scales.[1] We have seen that a fermion zero mode emerges owing to a special structure of the SU(2) slave-boson theory. Furthermore, we demonstrated how the presence of such vortices can allow the doping independent decreasing ratio of superfluid weight.

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[12] Inserting the following expression $h_i^{SF} = (\sqrt{x} + \delta b_i)\begin{pmatrix} z_{i1} \\ -i(-1)^{i_x+i_y} z_{i2} \end{pmatrix}$ with $z_{i1} = e^{i\alpha_i} e^{-i\frac{\phi_i}{2}} \cos\frac{\theta_i}{2}$ and $z_{i2} = e^{i\alpha_i} e^{i\frac{\phi_i}{2}} \sin\frac{\theta_i}{2}$ into the holon sector of the SU(2) slave-boson theory Eq. (5), where this representation can be derived easily from an SU(2) rotation of $h_i^{SF} = g_i^\dagger h_i^{dSC}$ with an SU(2) matrix field $g_i^\dagger = \exp\Big[-i(-1)^{i_x+i_y}\frac{\theta_i}{2}\tau_1\Big]\exp\Big[-i\frac{\phi_i}{2}\tau_3\Big]$ and $h_i^{dSC} = \begin{pmatrix} \sqrt{x} \\ 0 \end{pmatrix}$, and performing integration of amplitude fluctuations δb_i, we find an effective field theory for the holon sector in Eq. (6).[8]

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